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Spatially-Limited Altruism, Mixed Clubs, and Local Income Redistribution

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Abstract

This paper extends the work of Pauly (1973) by analysing the optimal organization of an economy in which individuals experience spatially-limited altruism. With such altruism, the nonpoor members of society care more about the poor living near them than about those living farther away. The main theme of the paper is that while the proximity of the poor gives mixed communities an altruistic advantage over homogeneous communities, the intermixing of rich and poor generates an efficiency loss in that public consumption in mixed communities cannot be tailored to suit individual preferences. As a result, a mixed community configuration (where income redistribution proceeds through local transfers) may or may not be superior to a homogeneous configuration (in which redistribution is conducted by the federal government). In addition to analysing this altruism/efficiency loss trade-off, the paper characterizes equilibrium outcomes when communities are organized by competitive developers.

Spatially-Limited Altruism, Mixed Clubs, and Local Income Redistribution

by

Jan K. Brueckner and Kangoh Lee*

1. Introduction

In an important paper, Pauly (1973) introduced the notion of spatially-limited altruism into the formal public finance literature. With this type of altruism, the rich care more about the poor living in their own community than about those living farther away. Pauly used this concept to argue that a policy of national income redistribution is inefficient. The reason is that such a policy (in Pauly's view) imposes a uniform redistributive standard on dissimilar local jurisdictions, each of which will have a different taste for redistribution when altruism is spatially limited.[1] This argument challenged the standard orthodoxy, which claimed that income redistribution is the responsibility of the national government.[2]

The present paper addresses a more comprehensive question within Pauly's framework by analysing the optimal spatial organization of an economy with spatially-limited altruism. Since he assumed a fixed distribution of the population, this issue did not arise in Pauly's analysis. The central question concerns the optimal spatial grouping of the rich and poor: should the rich and poor live together in mixed communities or should they live separately in homogeneous communities? The answer to this question bears on the assignment of income redistribution duties since redistribution can proceed through local transfers when communities are mixed while intercommunity transfers (and thus a national redistribution program) are needed when communities are homogeneous.

Although a direct application of Pauly's model would suggest that communities should be mixed so that the rich can experience altruism most intensely, the issue is more complex in the present framework. This is a consequence of the key additional assumption that individuals in the economy consume local public goods, which was not present in Pauly's analysis. Since public consumption in mixed communities cannot be tailored to suit individual preferences, the altruistic benefits of mixed communities are accompanied by an efficiency loss on the consumption side. This loss is avoided in homogeneous communities but enjoyment of altruism is sacrificed. The presence of this trade-off makes the optimal spatial structure of the economy indeterminate in general. A major goal of the analysis is to resolve this indeterminacy by identifying conditions under which a mixed community configuration is optimal. The paper also analyses equilibrium community configurations under the assumption that communities are organized by competitive, profit-maximizing developers. It is shown (subject to certain qualifications) that mixed communities emerge in equilibrium whenever such a configuration is desirable from an efficiency standpoint.

The paper's lessons about the optimality of local redistribution are somewhat different than in Pauly's analysis. Since the national government can always duplicate local transfers, assignment of the redistributive function to the national level is never inefficient. However, institutional parsimony suggests that redistribution should be carried out by local governments whenever this is feasible (that is, when the economy is organized in identical mixed communities). It should be noted that had

Pauly considered the identical community case (which emerges when the community structure is optimized), his indictment of national redistribution would have been softened along the above lines.

The paper's analytical framework is based on the standard economic model of clubs, as developed by Buchanan (1965), Berglas (1976b), and Berglas and Pines (1981).[3] Although the connection to Pauly makes the local redistribution question important, the paper's main contribution is in fact to extend the theory of clubs by stating new conditions under which mixed clubs are optimal. There has been considerable interest in this issue in the literature. Berglas (1976a), for example, showed that mixing is desirable when different types of people are complementary in production. More recently, Berglas (1984) proved the less obvious result that mixing may be optimal in the presence of multiple public goods. In analysing spatially-limited altruism, this paper identifies a new force favoring the formation of mixed clubs.

2. Normative Analysis

The model has two types of individuals, a and b, with the a's feeling altruism for the b's, as specified further below. The b's comprise a fraction θ of the economy's total population N, with the a's accounting for $1-\theta$ of the total. Exogenous incomes for the two groups are I^a and I^b. Given the a's altruism, it is natural to suppose that the b's are relatively poor ($I^b < I^a$), although this assumption plays no role in the analysis. Consumption goods in the economy include a private good x and a congested public good z. The cost in terms of x of providing public consumption z to a community (hereafter "club") of n people is C(z,n). C is increasing and convex in z, and congestion implies that the partial derivative C_n is positive. A further assumption is that for any z > 0, per capita cost $C(z,n)/n$ is a U-shaped function of n, which guarantees the existence of a positive finite optimal club size.

The (well-behaved) type-a and type-b utility functions are $U(x,z,k)$ and $V(x,z)$, where the k argument captures the altruism felt by the a's. A fundamental assumption is that this altruism is spatially limited, which means that an a-type cares more about the b's living in his own club than those living in other clubs. Moreover, the possibility of joint consumption of public goods means that k does not depend simply on the post-transfer income of the b's (as in the usual formulation of altruism) but instead reflects their achieved utility level. There are various ways of modeling the spatially-limited utility interdependence implied by these assumptions. One possibility would be to assume that k equals a weighted sum of the utility levels of type-b consumers, with a higher weight applied to b's in the home club of a representative a-type. To formalize this notion, let the home club have population n and a type-b proportion of σ, so that it contains σn b-types. Then k would equal $[(\alpha + \beta)\sigma n v^{home} + \beta(\theta N - \sigma n)v^{away}]$, where v^{home} is the type-b utility in the home club, v^{away} is the (average) type-b utility in other clubs, $\beta \geq 0$ is a parameter measuring "generalized" altruism (which is felt regardless of the location of the b's), and $\alpha \geq 0$ is parameter measuring "local" altruism (which is felt only toward b's in the same club). Recall from above that θN equals the number of b-types in the economy. While this is in some ways a natural formulation, it has the peculiar implication that for a given uniform type-b utility (v^{home}=v^{away}=constant), k is increasing in σn, implying that an a-type is happier in a club with a larger type-b population. This seems inconsistent with typical behavior, under which the nonpoor care about the welfare of the poor but do not wish to live surrounded by them. A modification that addresses this objection would be to write the v^{home} term above as $[\alpha g(\sigma n) + \beta\sigma n]v^{home}$, where g is a function satisfying $g(\sigma n) \geq 0$, $g(0) = 0$, and $g'(0) > 0$. This formulation allows k to rise initially with the type-b population when $v^{home}=v^{away}$, but the possibility that g' could turn negative means that further increases may ultimately depress k in a realistic fashion. Since the appearance of the absolute population size σn in the g function is inconvenient in the later analysis, this function is replaced by a function $f(\sigma)$ that depends only on the type-b proportion σ. This yields $k = (\alpha f(\sigma) + \beta\sigma n)v^{home} + \beta(\theta N - \sigma n)v^{away}$, where f satisfies $f(\sigma) \geq 0$, $f(0) = 0$, and $f'(0) > 0$. Again, f' could turn negative as σ increases, expressing an aversion on the part of the a's to living in a community with a high proportion of b's.

The normative problem is to characterize the Pareto-efficient club configurations in the model. As will be seen below, three types of configurations are potentially efficient. The first configuration, denoted H, consists of homogeneous type-a and type-b clubs. The second configuration contains only mixed clubs, each of which mirrors the overall composition of the population (having a type-b proportion equal to θ). This configuration is denoted CM, for "completely mixed." In the third configuration, denoted PM for "partially mixed," mixed clubs coexist with homogeneous clubs. As explained below, the choice between these three configurations can be expressed in the form of a single nonlinear programming problem. Before turning to this problem, however, it will be useful to compare the features of the H and CM configurations. This comparison will highlight the fundamental trade-off involved in the choice between homogeneous and mixed clubs.

Club configuations in model must satisfy the standard requirement of horizontal equity (identical utilities for identical people). Moreover, all clubs of a given type (mixed, homogeneous type-a, homogeneous type-b) are constrained to be identical (configurations violating this requirement are inefficient).[4] The first step in analysing the H configuration is to note that since type-b utilities are uniform under the horizontal equity requirement, the altruism expression k is evaluated with $v^{home}=v^{away}=v$. Furthermore, since clubs are homogeneous, the type-b proportion σ equals zero in each club where the a-types live. Recalling that $f(0) = 0$, these facts mean that the altruism argument under H satisfies $k = \beta\theta Nv \equiv \delta v$, where v is the uniform type-b utility and $\delta \equiv \beta\theta N$. A Pareto-efficient H configuration then solves the following problem:

$$\max \; U(x^{ah},z^{a},\delta v)$$

$$\text{s.t. } V(x^{bh},z^{b}) = v \tag{1}$$

$$(1-\theta)Nx^{ah} + \theta Nx^{bh} + [(1-\theta)N/n^{a}]C(z^{a},n^{a}) + (\theta N/n^{b})C(z^{b},n^{b}) = (1-\theta)NI^{a} + \theta NI^{b}. \tag{2}$$

Eq. (2) above is the resource constraint for an economy with homogeneous clubs. The RHS is total income in the economy, and the first two terms on the LHS give total consumption of the private good x (the h superscripts indicate that the x values apply to homogeneous clubs). The remaining terms give the cost of public good provision in all the economy's clubs. Note that the number of clubs of each type equals group population [$(1-\theta)N$ for the a's, θN for the b's] divided by the relevant club population (n^{a} or n^{b}). As is standard in club theory, we ignore the fact that these expressions need not be integer-valued (the problem is inconsequential if N is large relative to optimal club sizes). The necessary conditions for an optimum in the above problem are the two constraints together with

$$n^a U_z/U_x = C_z^a \qquad (3)$$

$$n^b V_z/V_x = C_z^b \qquad (4)$$

$$C_n^i = C^i/n^i, \quad i=a,b, \qquad (5)$$

where subscripts denote partial derivatives and where the i superscripts on C and C_n indicate that the functions are evaluated at (z^i,n^i), i=a,b. Eqs. (3) and (4) are the Samuelson conditions for the two types of clubs, and (5) indicates that club populations are chosen to minimize the per capita cost of (optimal) public consumption.[5]

The key difference between the H and CM configurations is that, because of the proximity of the b's, the a's enjoy greater altruistic benefits than in the homogeneous clubs formed under H. This can be seen by computing the value of k in mixed clubs. Since $v^{home}=v^{away}=v$ and f is now evaluated at θ rather than zero, $k = [\alpha f(\theta) + \beta\theta n]v + \beta(\theta N - \theta n)v = [\alpha f(\theta) + \delta]v$, which exceeds the previous value of δv. A disadvantage of CM, however, is that in contrast to the H configuration, the public good level in its mixed clubs cannot be tailored to suit individual preferences. Substituting the new value of k, a Pareto-efficient CM configuration solves the following problem:

$$\max \ U(x^a,z,[\alpha f(\theta) + \delta]v)$$

$$\text{s.t. } V(x^b,z) = v \qquad (6)$$

$$(1-\theta)nx^a + \theta nx^b + C(z,n) = (1-\theta)nI^a + \theta nI^b. \qquad (7)$$

Eq. (7) is the resource constraint for a representative club, with n giving the club's population and z the common public good level consumed by its residents. The necessary conditions for an (interior) optimum are the two constraints along with[6]

$$(1-\theta)nU_z/U_x + \theta nV_z/V_x = C_z \tag{8}$$

$$C_n = C/n. \tag{9}$$

Eq. (8) is the Samuelson condition for the mixed club, which reflects the compromise of tastes imposed by heterogeneity, and (9) is the per capita cost-minimization condition. The number of clubs N/n need not be integer-valued, but this problem is again ignored.[7]

The key to comparing H and CM configurations for a common value of v is to note that the CM constraint (7) is equivalent to the H constraint (2) together with the side conditions

$$z^a = z^b, \quad n^a = n^b. \tag{10}$$

These "mixing constraints," which are necessary for common type-a and type-b consumption of public goods, reduce the size of the CM opportunity set relative to that of the H problem. Ordinarily, this would lead to a lower value of the objective function (type-a utility) under CM. However, since the altruistic advantage of mixed clubs means that, for given values of the choice variables, the CM objective function achieves a higher value than the H function, the effect of the smaller CM opportunity set may be reversed. Together, these considerations imply that the preferred configuration in a choice between H and CM cannot be determined in general.

Intuitively, this indeterminacy arises because there is a trade-off between the altruistic advantage of mixed clubs and the efficiency loss resulting from common consumption of public goods by people of different types. Clearly, this trade-off can be resolved in favor of mixed clubs if the altruistic advantage from mixing is sufficiently large or if the efficiency loss from mixing is sufficiently small. The magnitude of the altruistic advantage in the model is related to the size of the local altruism parameter α, and it is easy to show that CM is superior to H whenever α is sufficiently large. The efficiency loss from mixing can be evaluated by focusing on the mixing constraints (10). If these constraints are satisfied at the H solution, then the efficiency loss vanishes, and CM is preferred to H. Whether the mixing constraints hold exactly or approximately depends on preferences. If preferences for z and x are sufficiently close, then the efficiencly loss from mixing will be small and CM will be preferred to H, with H preferred otherwise. Rigorous versions of the above claims will be provided below as part of the general analysis of the choice between H, CM, and PM.

The choice between H and CM can be illustrated diagrammatically by drawing utility possibility frontiers for the two configurations. For a given configuration, the utility frontier shows the maximum achievable type-a utility (denoted u) for each value of v. Examples of the H and CM frontiers are illustrated in Figure 1. The presence of altruism means that both frontiers may contain upward-sloping segments, indicating that both utility levels can be raised simultaneously through appropriate redistribution. This follows because the slopes of the H and CM frontiers are $\delta U_k - \theta U_x/(1-\theta)V_x$ and $[\alpha f(\theta) + \delta]U_k - \theta U_x/(1-\theta)V_x$ respectively, either of which may be positive (the marginal utilities are evaluated, course at the appropriate solution, H or CM). For later convenience, the H frontier in Figure 1 is shown as downward sloping.[8]

As noted in the introduction, an important difference between the H and CM configurations is that the minimal institutional structure required to carry out income redistribution is different under the two regimes. Under the H configuration, each b-type receives a transfer equal to $x^b + C^b/n^b - I^b$ while each a-type provides a transfer equal to $I^a - (x^a + C^a/n^a)$ (the signs of these transfers are in fact unrestricted). Since the transfer payments cross club boundaries, it is clear that redistribution must be carried out by the federal government under the H configuration. Under the CM configuration, by contrast, each b-type receives a transfer of $x^{bh} + C/n - I^b$ while each a-type provides a transfer of $I^a - (x^{ah} + C/n)$ (this assumes that each person pays his share of public good costs). Although these transfers could be carried out by the federal government, the proximity of the a's and the b's means that this duty could just as well be assigned to the local government. Since local governments must exist in any case to provide the public good, this assignment is clearly desirable when it is feasible. While local redistribution is therefore the preferred system when the economy is organized in the CM configuration, an economy that relies on local redistribution cannot attain the H configuration.[9] For this reason, reliance on local redistribution can be inefficient in a world with spatially-limited altruism, in contrast to Pauly's finding.

With the above background, the programming approach to solving the Pareto-optimality problem is easily understood. Feasible club configurations in the general programming problem include H and CM as well as the PM configuration discussed above, where mixed and homogeneous clubs coexist. In this more complex setting, the horizontal equity requirement means that members of each group must enjoy the same utility level regardless of whether they live in a mixed or homogeneous club. In addition, all clubs of a given type must be identical, as before. Letting

Q denote the number of mixed clubs, σ denote the type-b proportion in their populations, and $I \equiv (1-\theta)I^a + \theta I^b$, the general Pareto-optimality problem can be written

$$\max \; U(x^a, z, [\alpha f(\sigma) + \delta]v)$$

$$\text{s.t. } V(x^b, z) = v$$

$$U(x^a, z, [\alpha f(\sigma) + \delta]v) = U(x^{ah}, z^a, \delta v) \qquad (11)$$

$$V(x^b, z) = V(x^{bh}, z^b) \qquad (12)$$

$$Q[(1-\sigma)nx^a + \sigma n x^b + C(z,n)] + [(1-\theta)N - Q(1-\sigma)n][n^a x^{ah} + C(z^a, n^a)]/n^a + [\theta N - Q\sigma n][n^b x^{bh} + C(z^b, n^b)]/n^b = NI. \qquad (13)$$

Note that (11) and (12) are the horizontal equity constraints and that $[(1-\theta)N - Q(1-\sigma)n]/n^a$ and $[\theta N - Q\sigma n]/n^b$ are the numbers of homogenous type-a and type-b clubs (total group size minus the population in mixed clubs divided by n^i, i=a,b).[10] Implicit constraints in the problem are $0 \le \sigma \le 1$ and $0 \le Q \le \min\{(1-\theta)N/(1-\sigma)n, \theta N/\sigma n\}$, with the last inequality saying that mixed clubs cannot contain more than the total population of either group.

As before, the first-order conditions for choice of the x, z, and n variables reduce to the mixed- and homogeneous-club Samuelson and per capita cost-minimization conditions. The Q and σ variables, however, are of more central interest since they determine the nature of the optimal club configuration. First, it is clear that the H configuration corresponds to $Q = 0$, while the CM configuration results from setting $\sigma = \theta$ and $Q = N/n$.[11] A PM configuration emerges when Q is positive and σ is different from θ. The following key result limits the class of admissible PM configurations:

Proposition 1. In a unique optimal configuration, mixed clubs can coexist with at most one type of homogeneous club (either a or b).

This result, which says that PM configurations can contain only one type of homogeneous club, follows directly from the fact that the Lagrangean expression L for the problem is linear in Q. This means that when the optimum is unique, it will be a corner solution, with Q either equal to zero or $\min\{(1-\theta)N/(1-\sigma)n,\ \theta N/\sigma n\}$ (alternatively, the optimal Q could be indeterminate). Since at least one of the equalities $(1-\sigma)nQ = (1-\theta)N$, $\sigma nQ = \theta N$ must therefore hold when the optimal Q is positive (and unique), it follows that the entire population of one or both groups fits into mixed clubs, as claimed.

Further consideration of the Lagrangean's Q-derivative (L_Q) gives insight into the conditions under which the optimal configuration contains mixed clubs. It is easily shown that L_Q has the same sign as

$$(1-\sigma)[(x^{ah} + C^a/n^a) - (x^a + C/n)] + \sigma[(x^{bh} + C^b/n^b) - (x^b + C/n)]. \tag{14}$$

Suppose that (14) evaluated at Q = 0 is positive for some σ.[12] Then, starting from configuration containing only homogeneous clubs, formation of mixed clubs is desirable since per capita consumption of resources can be reduced (with utility held constant) by moving individuals out of homogeneous clubs into a mixed club with the given σ. To see this, note that when (14) is positive, resource consumption by mixed-club residents, which equals $(1-\sigma)nx^a + \sigma nx^b + C$, is greater than their consumption in homogeneous clubs, which equals $(1-\sigma)n(x^{ah} + C^a/n^a) + \sigma n(x^{bh} + C^b/n^b)$ (dividing by n puts the comparison in per capita terms). If, on the other hand, (14) evaluated at Q = 0 is negative for all $0 < \sigma < 1$, then relocating homogeneous-club residents into any (equal-utility) mixed club raises per capita consumption of resources. In this case, it is clear that formation of mixed clubs is undesirable. This heuristic discussion illustrates the general condition for choice of Q: given the linearity of L in Q, the optimal Q is positive if (14) evaluated at Q = 0 is positive for some σ, while the optimal Q is zero if (14) evaluated at Q = 0 is negative for all $0 < \sigma < 1$.[13]

Since it may be shown that $x^{bh} + C^b/n^b$ is less than or equal to $x^b + C/n$, the second term in (14) is always nonpositive. However, the analogous comparison for the a's is indeterminate, which makes the first term in (14) (and thus the entire expression) ambiguous in sign. The first fact is due to the consumption inefficiency of mixed clubs, which means that resource consumption by the b's must be at least as great as in a homogeneous club offering the same utility. To see this formally, note that the satisfaction of the type-b Samuelson and per capita cost-minimization conditions guarantees that the b's in a homogeneous club achieve the required utility level v with the lowest possible per capita consumption of resources. Since type-b consumption in mixed clubs is determined by different first-order conditions, it follows that $x^b + C/n$ is at least as large as $x^{bh} + C^b/n^b$. While this effect is also present for the a's, the countervailing altruistic advantage of mixed clubs means that the resources required for the a's to achieve a given utility could be either higher or lower than in a homogeneous club. Note that for formation of mixed clubs to be (uniquely) optimal, the latter condition must hold, with $x^{ah} + C^a/n^a > x^a + C/n$ and type-a resource requirements lower in a mixed club. Otherwise, (14) cannot be positive.

Having analysed the general conditions under which mixed clubs are optimal, the next question concerns the choice between CM and PM. To address this question, start with the CM configuration, where $Q = N/n$ and $\sigma = \theta$, and consider the gains and losses from allowing σ to deviate from θ. First, some change in σ will typically make each mixed club's population makeup more advantageous from an altruistic point of view. Recalling that local altruism depends on the type-b proportion through the function f, the a's will gain from increasing (decreasing) the type-b proportion relative to θ as $f'(\theta)$ is positive (negative). This altruistic gain has a cost, however, in that the individuals displaced from the mixed club must be guaranteed the same utility as those that remain. If the amount of extra resources required to achieve this equality is less than the (appropriately measured) altruistic gain, then some deviation of σ away from θ is desirable.

To formalize these considerations, the appropriate σ-derivative is computed by substituting $Q = \min\{(1-\theta)N/(1-\sigma)n, \theta N/\sigma n\}$ into the Lagrangean expression and differentiating in the separate cases where $\sigma \geq \theta$ and $\sigma \leq \theta$. The derivative in the first case, denoted $L_\sigma|_{\sigma\geq\theta}$, has the same sign as

$$\alpha U_k f'(\sigma)v/U_x - [(x^{ah} + C^a/n^a) - (x^a + C/n)]/\sigma(1-\sigma) \quad (15)$$

and the derivative in the second case, $L_\sigma|_{\sigma\leq\theta}$, has the same sign as

$$\alpha U_k f'(\sigma)v/U_x + [(x^{bh} + C^b/n^b) - (x^b + C/n)]/(1-\sigma)^2. \quad (16)$$

Suppose that (15) evaluted at $\sigma = \theta$ is positive or that (16) evaluated at $\sigma = \theta$ is negative (or both). Then there exist PM configurations close to CM that yield higher values of the objective function than CM itself, establishing that CM cannot be optimal. To relate this result to the previous intuitive discussion, note that the altruistic gain from increasing σ above θ (which could be negative) is captured by the first term in (15) while the resource cost discussed above is the second term. The cost of displacing an a-type into a homogeneous club equals the difference between per capita consumption of the a's in homogenous and mixed clubs, and the factor $1/\sigma(1-\sigma)$ apportions these costs among the a-types remaining in the mixed club.[14] For (15) to be positive, indicating that PM configurations with σ slightly above θ are preferable to CM, the altruistic gain in the first term must exceed the displacement cost in the second term. Similarly, for (16) to be negative (meaning that σ's slightly below θ are preferred to θ), the altruistic gain from decreasing σ (the negative of the first term) must exceed the cost of displacing a b-type into a homogeneous club (the second term).

In the remainder of this section of the paper, we will prove three propositions that help identify the optimal club configuration under various conditions. The first proposition, which is based on eqs. (15) and (16), relates to the choice between CM and PM when H is not optimal. The last two propositions, which are based on (14), relate to the choice between H on the one hand and some mixed club configuration (CM or PM) on the other. The first result is as follows:

> **Proposition 2**. Suppose that $\alpha > 0$ and that $f'(\sigma) < 0$ holds for all $\sigma \geq \theta$. Then, if the optimal configuration contains mixed clubs, it must be a PM configuration with $\sigma < \theta$.

To prove this result, note first that since $x^{bh} + C^b/n^b \leq x^b + C/n$ from above, the cost of displacing a b-type into a homogeneous club is always nonpositive. Given that $f'(\theta)$ is negative by assumption, it then follows that (16) is negative at $\sigma = \theta$, so that there exist PM configurations with

$\sigma < \theta$ that are superior to CM. Intuitively, when $f'(\theta) < 0$, there are both resource savings and altruistic gains from relocating b-types into homogeneous clubs, making PM configurations with σ less than (but close to) θ superior to CM. To show further that no PM configuration with $\sigma > \theta$ can be optimal under the given assumptions, note first that (15) must equal zero at a PM optimum with $\sigma > \theta$. Next, recall from above that the first term in (14) (and hence the second term in (15)) must be positive at a mixed-club optimum (this is required to make (14) positive). But this fact, combined with the assumption that $f' < 0$ holds for $\sigma \geq \theta$, means that (15) is negative at any supposedly optimal PM configuration with $\sigma > \theta$. This contradiction establishes that such a configuration cannot be optimal.

It should be realized that when the conditions of Proposition 2 are not satisfied, little can be said about the location of a mixed club optimum. To see this, note that when $f'(\theta) > 0$, both (15) and (16) evaluated at $\sigma = \theta$ are ambiguous in sign, so that CM is not clearly inferior to some PM configuration. However, even when these expressions are respectively negative and positive (indicating that small deviations of σ away from θ are undesirable), a PM optimum is not ruled out. The reason is that $L_{\sigma\sigma}$ is of ambiguous sign on both sides of θ, which means that the objective function could reach higher values when σ is far from θ even though PM configurations close to CM are inferior to CM. Finally, although the relevant derivative ((15) or (16)) must be zero at the σ associated with a PM optimum, the ambiguity of $L_{\sigma\sigma}$ means that the derivative could also be zero at nonoptimal values of σ.

The remaining propositions of this section show how the choice between H on the one hand and CM or PM on the other depends on the magnitude of α and the dispersion of preferences. Recall that the earlier discussion claimed that CM is preferred to H when the altruistic gain from mixing (as measured by the size of α) is sufficiently large. The following proposition provides a more comprehensive result for the general choice problem:

> **Proposition 3**. For each type-b utility level v, there exists a critical value of α, denoted $\alpha^*(v) \geq 0$, such that H is optimal when $\alpha < \alpha^*(v)$ and either CM or PM is optimal when $\alpha > \alpha^*(v)$. If the mixing constraints (10) hold under H, then $\alpha^*(v) = 0$. Otherwise, $\alpha^*(v) > 0$.

The first step in proving this proposition is to establish that when local altruism is absent ($\alpha = 0$), the L_Q expression (14) evaluated at Q = 0 (denoted F hereafter) satisfies $F \leq 0$ for all σ between zero and one.[15] To see this, note that when $\alpha = 0$, there is no countervailing force to offset the efficiency loss of mixed clubs from the perspective of the a's. From the above discussion, this implies that $x^i + C/n \geq x^{ih} + C^i/n^i$ holds for i = a,b, yielding $F \leq 0$. More precisely, it can be shown that F = 0 holds when $\alpha = 0$ and the mixing constraints (10) are satisfied under H and that F < 0 holds when $\alpha = 0$ and these constraints are not satisfied. The first claim follows from the fact that the equalities $x^a = x^{ah}$, $x^b = x^{bh}$, $z = z^a = z^b$, and $n = n^a = n^b$ hold when (10) is satisfied. To see this fact, note that since the homogeneous-club Samuelson conditions imply the mixed-club Samuelson condition (multiply (3) and (4) by $(1-\sigma)$ and σ respectively and add), the mixed-club condition holds at the common homogeneous-club solution values. Since (5) and (9) are the same and utility levels must be equal between mixed and homogeneous clubs, it follows that x^{ah}, x^{bh}, and the common homogeneous-club z and n values satisfy the mixed-club conditions, implying that the two solutions are identical and thus that F = 0. In the alternate case where the mixing constraints do not hold, the efficiency loss from mixing reasserts itself and it follows that resource requirements are higher in mixed than in homogeneous clubs, implying $F < 0$.

The next step in the proof of Proposition 3 is to note that the derivative of F with respect to α, denoted F_α, is positive for any σ between zero and one (this fact is established in the appendix). Since it has been shown that $F = 0$ when $\alpha = 0$ and the mixing constraints are satisfied, positivity of F_α then means that $F > 0$ holds for all positive α in this case. This proves that CM or PM is optimal for all $\alpha > 0$ when the mixing constraints are satisfied (implying that the critical value $\alpha^*(v)$ equals zero). When the mixing constraints are not satisfied, $F < 0$ holds for any σ when $\alpha = 0$, and since $F_\alpha > 0$, there exists a critical α value $\alpha^{**}(\sigma,v) > 0$ that depends on σ and v such that F is negative (positive) for α less than (greater than) $\alpha^{**}(\sigma,v)$. It then follows that H is optimal when $\alpha < \inf\{\alpha^{**}(\sigma,v) | \ 0 < \sigma < 1\}$ since α will then be small enough to make F negative regardless of the value of σ (recall that this is required for H to be optimal). When $\alpha > \inf\{\alpha^{**}(\sigma,v) | \ 0 < \sigma < 1\}$, on the other hand, $F > 0$ will hold for some σ, and CM or PM will be optimal. Setting $\alpha^*(v) \equiv \inf\{\alpha^{**}(\sigma,v) | \ 0 < \sigma < 1\}$ establishes the proposition.

An implication of Proposition 3 is that CM or PM is optimal if the mixing constraints are satisfied. Although it is possible to construct pathological examples where (10) holds under H, this outcome arises naturally when preferences for x and z are the same. Suppose, for example, that $U(x,z,k) \equiv V(x,z) + W(k)$, so that k enters the type-a utility function in an additively separable manner and the (x,z) portion of the function is identical to the type-b utility function. Then it is easy to see that the mixing constraints hold under H when income is redistributed so as to achieve identical post-transfer incomes (equal to $(1-\theta)I_a + \theta I_b$) for the two groups. For the v value corresponding to this post-transfer income, CM or PM will be optimal. Suppose further that U satisfies the above assumptions and in addition utility is transferable, with $V(x,z) \equiv x + S(z)$, where $S' > 0$ and $S'' < 0$. Then, since conditions (3)-(5) do not involve x, they yield the same common (z,n) solution for all values of v. As a result, (10) holds and CM or PM is optimal for all v in this case.

While the absence of an efficiency loss from mixing makes some type of mixed-club configuration optimal, it is intuitively clear that a sufficiently small efficiency loss will lead to the same result. Moroever, as noted above, the size of the loss is related to the divergence in preferences between the a's and the b's. If preferences are "similar," then the efficiency loss will be small, while if preferences are quite dissimilar, then the loss will be large. The following result draws a precise connection between the extent of divergence in preferences and the identity of the optimal configuration in the transferable utility case:

<u>Proposition 4</u>. Suppose that $U(x,z,k) \equiv x + \Omega S(z) + W(k)$ and $V(x,z) \equiv x + S(z)$, with $S' > 0$, $S'' < 0$, and $\Omega \geq 0$. Then when $\alpha > 0$, there exist numbers $\Omega_1^*(v) > 1$ and $0 \leq \Omega_2^*(v) < 1$ such that CM or PM is optimal at a given v when Ω satisfies $\Omega_2^*(v) < \Omega < \Omega_1^*(v)$ and H is optimal when $\Omega > \Omega_1^*(v)$ or $\Omega < \Omega_2^*(v)$.

This result says that when preferences are sufficiently close in the transferable utility case, CM or PM is optimal (with H optimal otherwise). The first step in proving the proposition is to note from above that when $\Omega = 1$, preferences are identical and the mixing constraints are satisfied for all v under H. From above, this implies that $F > 0$ (making CM or PM optimal). The next step is to note that $F_\Omega > 0$ holds when $\Omega < 1$ and that $F_\Omega < 0$ holds when $\Omega > 1$ (this is shown in the appendix). These facts imply that for given v and σ, there exists a critical Ω value $\Omega_1^{**}(\sigma,v) > 1$ such that F is greater than (less than) zero when Ω is greater than one but less than (greater than) $\Omega_1^{**}(\sigma,v)$. Similarly, if $F < 0$ holds for $\Omega = 0$, then there exists a positive critical Ω value $\Omega_2^{**}(\sigma,v) < 1$ below which F changes sign from positive to negative. If $F \geq 0$ at $\Omega = 0$, then a positive critical value does not exist and $\Omega_2^{**}(\sigma,v)$ is set equal to zero. Recalling that CM or PM is optimal if F is positive for some σ, it follows that CM or PM is optimal when Ω satisfies $\inf\{\Omega_2^{**}(\sigma,v)|\ 0 < \sigma < 1\} < \Omega < \sup\{\Omega_1^{**}(\sigma,v)|\ 0 < \sigma < 1\}$ since Ω will then lie in the range where F is positive for some σ. Conversely, when $\Omega > \sup\{\Omega_1^{**}(\sigma,v)|\ 0 < \sigma < 1\}$ or $\Omega < \inf\{\Omega_2^{**}(\sigma,v)|\ 0 < \sigma < 1\}$, F is negative for all σ and H is optimal (note that if $\Omega_2^{**}(\sigma,v) = 0$ for all σ, then H is never optimal for $\Omega < 1$). Setting $\Omega_1^*(v) \equiv \sup\{\Omega_1^{**}(\sigma,v)|\ 0 < \sigma < 1\}$ and $\Omega_2^*(v) \equiv \inf\{\Omega_2^{**}(\sigma,v)|\ 0 < \sigma < 1\}$ establishes the propostion.

Note that by combining Propositions 2-4, sufficient conditions for the optimality of PM can be stated. Clearly, when $f'(\sigma) < 0$ for $\sigma \geq \theta$ and $\alpha > \alpha^*(v)$, then PM is optimal at the given v. If in the transferable utility case, $f' < 0$ again holds for $\sigma \geq \theta$ and $\Omega_2^*(v) < \Omega < \Omega_1^*(v)$, then PM is once again optimal. It should be noted that since simple sufficient conditions for CM to dominate PM are not available, it is not possible to state simple conditions under which CM is optimal.

The general choice problem can be illustrated diagrammatically by drawing a utility frontier based on the solution to the general programming problem. When either H or CM is optimal at a given v, the general frontier coincides with either the H or CM frontier of Figure 1 at that v. If, however, PM is optimal, then the general frontier passes above both the H and CM frontiers at the given v.

A final point concerns the assignment of income redistribution responsibilities in the general model. While it was pointed out above that local governments should handle income redistribution under the CM configuration, this assignment is not generally feasible under the PM configuration since interclub transfers will typically be required. Therefore, even though mixed clubs exist under PM, a federal redistribution system must exist to support such a configuration.

3. Positive Analysis

The key feature of standard models of altruism is that through the voluntary action of individuals in the economy, the utility of the poor group is raised above the level corresponding to the original distribution of income. The purpose of the positive analysis in this section of the paper is to see whether this outcome obtains in the present model. The goal is to determine the level of v that actually emerges as a result of decentralized behavior (v, of course, was parametric in the planning problem). The analysis is carried out under the assumption that clubs are organized by competitive developers, as in Berglas (1976b) and Berglas and Pines (1980, 1981). A key feature of the competitive model is that developers, being small operators, are not able to make interclub transfers. This means that PM configurations and all H configurations except one are not attainable in the model. The only feasible configurations are the CM configuration and the H configuration based on the original distribution of income, neither of which involves interclub transfers. The unattainability of some club configurations means that equilibrium in the model may not be efficient. To reduce the likelihood of this outcome, one possible source of inefficiency is removed by the assumption that generalized altruism is absent ($\delta = 0$), which means that the H frontier in Figure 1 is always downward sloping. This rules out a situation in which the one attainable H configuration, denoted NR for "no redistribution", is automatically Pareto-inefficient as a result of being dominated by one of the unattainable configurations on the H frontier (this configuration is shown in Figure 1).

A critical additional assumption is that in forming mixed clubs, developers are required (by law, perhaps) to mix the a's and the b's in accordance with the overall composition of the population, forming clubs with type-b proportions σ equal to θ. This requirement will be referred to as the "club composition constraint." As will be seen below, equilibrium may not exist when developers are allowed to choose σ. In order to satisfy this requirement, as well as to carry out some of the actions described below, developers must obviously be able to identify individuals by type.

The analysis first derives the features of the homogenous and mixed clubs organized by developers. The discussion then identifies the club structure (homogeneous or mixed) that actually emerges in equilibrium. Consider the problem faced by a developer organizing a homogeneous type-a club (an exactly parallel argument applies to the formation of type-b clubs). The developer charges a club entry fee denoted by P^a while choosing the public good level in the club and the size of its population. Suppose the developer wishes to guarantee his club members a utility level equal to u. Recalling that $\delta = 0$ and that homogeneous clubs must reflect the original distribution of income, P^a must then satisfy $U(I^a-P^a,z^a,0) = u$. This equation implicitly defines P^a as a function of z^a and u, with $P^a_z = U_z/U_x$ and $P^a_u = -1/U_x < 0$. The developer's profit can then be written

$$\pi^a = n^a P^a(z^a, u) - C(z^a, n^a). \qquad (17)$$

For given u, the developer chooses z^a and n^a to maximize (17), with the first-order conditions being the Samuelson condition (3) and $P^a - C_n^a = 0$. The realized profit level depends on the parametric utility u. It is easy to see that when u assumes the value corresponding to the NR point in Figure 1, profit equals zero. Since P^a (and hence π^a) is decreasing in u, profit is then positive (negative) for u below (above) the NR value. To see the first claim, note that when $\pi^a = 0$, the condition $P^a - C_n^a = 0$ reduces to the per capita cost-minimization condition (5). Since the Samuelson condition also holds and since the zero profit condition together with the budget constraint $x^a + P^a = I^a$ implies satisfaction of the club resource constraint $n^a x^a + C^a = n^a I^a$, it follows that the planning conditions and club equilibrium conditions are identical. This means that the zero-profit u is the same as the u value achieved at the no-redistribution point in the planning problem, as claimed.

Now consider the problem of the mixed-club developer, who provides a common level of the public good to both the a's and the b's. While the a's may voluntarily enter a mixed club to benefit from the presence of the b's, the b's themselves have no such incentive and will require compensation in the form of a transfer payment to join a mixed club. The transfer T is provided by the developer, who collects the necessary funds from the a's. While the two groups pay a common club entry fee P, the presence of the transfer makes their net costs of joining the club different.

To guarantee utilities of u and v to the type-a and type-b members of his club, the developer must choose P and T to satisfy the following conditions:

$$U(I^a - P - \theta T/(1-\theta), z, \alpha f(\theta)v) = u \qquad (18)$$

$$V(I^b - P + T, z) = v. \qquad (19)$$

Note in (18) that the per capita tax on the a's depends on the type-b proportion, which is set at θ to satisfy the club composition constraint.[16] Total differentiation of (18) and (19) shows that $P_z = (1-\theta)U_z/U_x + \theta V_z/V_x$ and that P is decreasing in u.[17] As before, the developer chooses z and n to maximize

$$\pi = nP(z,u,v) - C(z,n), \qquad (20)$$

with first-order conditions being the Samuelson condition (8) and $P - C_n = 0$. The realized profit level again depends on the parametric utilities u and v. It is easily seen that for given v, profit is zero when u equals the value on the CM frontier at that v. Profit is positive (negative) when u is below (above) the CM-frontier value. As before, the first claim follows because when $\pi = 0$, the condition $P - C_n = 0$ reduces to the per capita cost-minimization condition (9) and the type-a and type-b budget constraints imply satisfaction of the club resource constraint. This means that for given v, the zero profit u is the same as the one achieved in the planning problem.

Having looked separately at homogeneous and mixed clubs, it is now possible to analyse the club structure that emerges in equilibrium. A club configuration will be an equilibrium in the model if its associated utility pair (u_E, v_E) has the following properties: i) club developers earn nonegative profit in the given configuration; ii) any type-a homogeneous club offering $u > u_E$ earns negative profit; iii) any type-b homogeneous club offering $v > v_E$ earns negative profit; iv) any mixed club offering utilities (u,v) such that $u \geq u_E$ and $v \geq v_E$ and at least one inequality holds strictly earns negative profit; v) the given club configuration accommodates the economy's population. In other words, an equilibrium club configuration must house the economy's population and there must exist no alternative clubs that are viable (that can attract residents by offering utilities higher than those enjoyed in the given configuration) and profitable (earning at least a zero profit for the developer). With this definition[18] in mind, the following result can be established:

> Proposition 5. Assume that generalized altruism is absent and that clubs are organized by competitive developers who are subject to the club composition constraint, as described above. If there are no points on the CM frontier satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$, where u_{NR} and v_{NR} are the utility levels at point NR on the H frontier, then the equilibrium club configuration is the homogeneous configuration corresponding to NR. If the CM frontier contains points satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$, with at least one equality holding strictly, then the set of equilibria consists of all points on the CM frontier satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$ and not Pareto-dominated by some other point.

The first step in proving this result is to establish that an equilibrium configuration cannot contain a homogeneous club offering a utility different from the NR level or a mixed club offering a utility pair not on the CM frontier. To see the first claim, recall that a homogeneous club offering a utility level above the NR level earns a negative profit. While a homogeneous type-a club offering $u = u' < u_{NR}$ earns a positive profit, such a club also cannot exist in equilibrium because there is an alternative homogeneous club offering $u = u' + \tau < u_{NR}$ (with $\tau > 0$) that is both viable and profitable (the same argument applies to type-b clubs). The second claim is proved similarly. Mixed clubs offering utility pairs above the CM frontier earn negative profits, while a mixed club offering a utility pair (u'',v'') below the CM frontier is ruled out because an alternative mixed club offering a utility pair ($u'' + \tau', v''$) below the CM frontier but with $\tau' > 0$ is both viable and profitable.

With the class of candidates for equilibrium thus narrowed, consider the first case in the proposition, where no points on the CM frontier satisfy $u \geq u_{NR}$ and $v \geq v_{NR}$. In this case, any mixed club that is a candidate for equilibrium must yield a utility level lower than the NR level for at least one group. Suppose that such a club has type-a utility equal to $u' < u_{NR}$. Since a homogeneous type-a club offering a utility u'' between u' and u_{NR} would attract away the a's and earn its developer a positive profit, it follows that the given mixed club cannot exist in equilibrium. With an equilibrium mixed club configuration ruled out by this argument, it remains to show that the homogeneous club configuration corresponding to NR satisfies the requirements of equilibrium in that viable and profitable alternative clubs do not exist. First, any viable alternative homogeneous club is unprofitable since it must offer a utility higher than the NR level to one group. Second, any viable alternative mixed club is also unprofitable since such a club must also improve on the NR utilities and thus must lie above the CM frontier. These observations complete the proof of the first half of the proposition.[19]

Now suppose that the CM frontier contains points satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$, with at least one equality holding strictly. In this case, the argument used above to rule out mixed clubs shows that all CM points with at least one utility less than the NR level are not equilibria (viable and profitable alternative homogeneous clubs exist). Furthermore, the homogeneous NR configuration cannot be an equilibrium because there exist viable alternative mixed clubs earning at least a zero profit. This leaves points on the CM frontier satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$ as candidates for equilibria. Suppose one of these points with coordinates (u',v') is Pareto-dominated by another such point with coordinates (u",v"), a possiblity given that the CM frontier may contain upward-sloping segments. In this case, (u',v') cannot be an equilibrium because an alternative mixed club offering v = v" and a u slightly less than u" would be viable and profitable. The remaining undominated points, however, satisfy the requirements of equilibrium. First, since any viable alternative homogeneous club must offer its group a utility above the NR level, such a club will lose money. In addition, since each equilibrium candidate is Pareto-undominated, any viable alternative mixed club must lie above the CM frontier (and thus must be unprofitable). This completes the proof of the second half of the proposition.

Note that, for simplicity, Proposition 5 does not cover the case where the CM frontier crosses the H frontier at NR without passing to the northeast of NR. In this case, it is easy to see that the CM and H configurations corresponding to NR are both equilibria. The second part of Proposition 5 is illustrated in Figure 1, where the CM frontier is shown passing to the northeast of NR. The Pareto-undominated CM points in this range, which comprise the set of equilibria, are contained in the segment JG of the frontier.

This analysis shows that when CM configurations exist that are Pareto-superior to the status-quo point NR, decentralized behavior drives the economy to one of these configurations. Since it is easily seen that the transfer T received by the b's is positive in any such equilibrium, the outcome is identical to that in a standard altruism model, where one group voluntarily relinquishes income to help the other.[20]

If CM is always preferred to PM, then equilibrium in the model is efficient in that no alternative club configuration is Pareto-superior to any equilibrium configuration (this is clear from Figure 1). However, if PM is sometimes superior to CM, then the general utility frontier will sometimes lie above the CM frontier. In this case, both the mixed- and homogeneous-club equilibria of the model may be inefficient (the general frontier could pass above either type of equilibrium point). This potential inefficiency is a consequence of the inability of competitive developers to make interclub transfers.

A difficulty with the conclusions of this analysis is that they depend critically on the presence of the club composition constraint, which might be viewed as an unrealistic requirement in a decentralized economy. Without this constraint, σ becomes a choice variable of the developer. The entry fee P now explicitly includes σ as an argument, and profit maximization requires $nP_\sigma = 0$. This condition reduces to

$$(1-\sigma)^2 U_k \alpha f'(\sigma) v/U_x - T = 0. \qquad (21)$$

Unless (21) holds at $\sigma = \theta$, the previous mixed-club equilibria lose their equilibrium status since developers can find profitable and viable alternative clubs with σ's different from θ. Under these circumstances, it can be shown that equilibrium may fail to exist, a fact which highlights the critical role of the club composition constraint.[21]

As a final point, it is interesting to ask how the economy would be organized if the a-types could specify the club configuration. Under such an arrangement, the goal of the a's would be to maximize their own utility subject to the constraint that the b's are willing to participate in the chosen configuration. Formally, this problem amounts to finding the point on the general utility frontier that maximizes u subject to the requirement that v exceeds or equals v_{NR}. Unless this latter constraint is satisfied, the b's will decline to participate in the chosen configuration, retreating instead to homogeneous clubs based on the original distribution of income. A difficulty with this choice process, however, is that cannot be viewed as decentralized.

4. Conclusion

This paper has analysed optimal club configurations in a model with spatially-limited altruism and has generated a number of results that are new to club theory. The paper has also exposed the trade-off that must be confronted in a evaluating a policy of local income redistribution. This is an important contribution because local redistribution is practiced in the U.S. on a variety of different levels. For example, unequal sharing of the costs of running school districts and providing other public services leads to extensive implicit redistribution among households at the community level. Moreover, the fact that state contributions to federal welfare programs are substantial and not at all uniform shows that the welfare system involves an important element of local redistribution.

What can be said about such policies on the basis of the discussion in this paper? The main practical lesson of the paper is that while local redistribution may be consistent with efficiency in the presence of spatially-limited altruism, the pursuit of such policies could involve a substantial welfare cost. Since there is no reason to think that real world economy mimics the developer model in avoiding undesirable equilibria, the economy could conceivably benefit from homogenization of communities and reassignment of the redistributive function to higher levels of government. While such a conclusion follows from the model, it cannot be taken literally as a policy prescription. The main reason is that complementarities in production (as in Berglas (1976a)) are probably important enough in the real world to invalidate any call for community homogenization based on public-sector considerations. In spite of this, awareness of the potential welfare loss from pursuit of local redistribution can only be beneficial in the analysis of policy questions related to fiscal federalism.

Appendix

This appendix first establishes the fact that $F_\alpha > 0$, which was used in the proof of Proposition 3. Differentiating (14), F_α equals

$$(1-\sigma)\{[x_\alpha^{ah} + (C_z^a/n^a)z_\alpha^a] - [x_\alpha^a + (C_z/n)z_\alpha]\} \qquad (A1)$$
$$+ \sigma\{[x_\alpha^{bh} + (C_z^b/n^b)z_\alpha^b] - [x_\alpha^b + (C_z/n)z_\alpha],$$

where the α subscripts denote partial derivatives. This expression can be simplified by differentiating the utility constraint (1) and using (4), which shows that the first term following σ in the second line equals zero. Furthermore, differentiating (12) yields $x_\alpha^b = -(V_z/V_x)z_\alpha$ and differentiating (11) yields

$$x_\alpha^a = (U_x^h/U_x)[x_\alpha^{ah} + (U_z^h/U_x^h)z_\alpha^a] - (U_k/U_x)f(\sigma)v$$
$$- (U_z/U_x)z_\alpha, \qquad (A2)$$

where the h subscripts on U_x and U_z indicate that these derivatives apply to homogeneous clubs. Substituting these expressions in (A1) and using the mixed-club Samuelson condition ((8) with σ replacing θ), all the terms involving z_α drop out and (A1) reduces to an expression with the same sign as

$$[(U_x/U_x^h) - 1](U_x^h x_\alpha^{ah} + U_z^h z_\alpha^a) + U_k f(\sigma)v. \qquad (A3)$$

The term in parentheses in the middle of (A3) is the change in utility in a homogeneous club resulting from a change in α, which must equal the change in utility in a mixed club by (11). But by the envelope theorem, the change in utility in a mixed club is just the partial derivative of the Lagrangean expression with respect to α, or $(1 + \mu)U_k f(\sigma)v$, where μ is the multiplier associated with (11). Substituting this expression in place of the middle term in (A3) and simplifying, (A3) reduces to an expresssion with the sign of $(1 + \mu)U_x/U_x^h - \mu$. Since it is easily seen from the first-order conditions for x^a and x^{ah} that $-1 < \mu < 0$, it follows that this expression is positive, establishing $F_\alpha > 0$.

Our next task is to establish the fact that $F_\Omega > (<) 0$ as $\Omega < (>) 1$, which was used in the proof of Proposition 4. The first step is to differentiate (14) with respect to Ω, which leads to the expression (A1) with Ω replacing α. Differentiating (11) and (12) with respect to Ω under transferable utility, substituting, and using the Samuelson condition, the modified (A1) reduces to $(1-\sigma)[S(z) - S(z^a)]$. Again using the Samuelson condition, it is easy to see that $z > (<) z^a$ holds as $\Omega < (>) 1$ (the mixed-club z is greater than (less than) the type-a value when the type-a demand for z is lower (higher) than the type-b demand). Using this fact to sign the above expression gives the desired result.

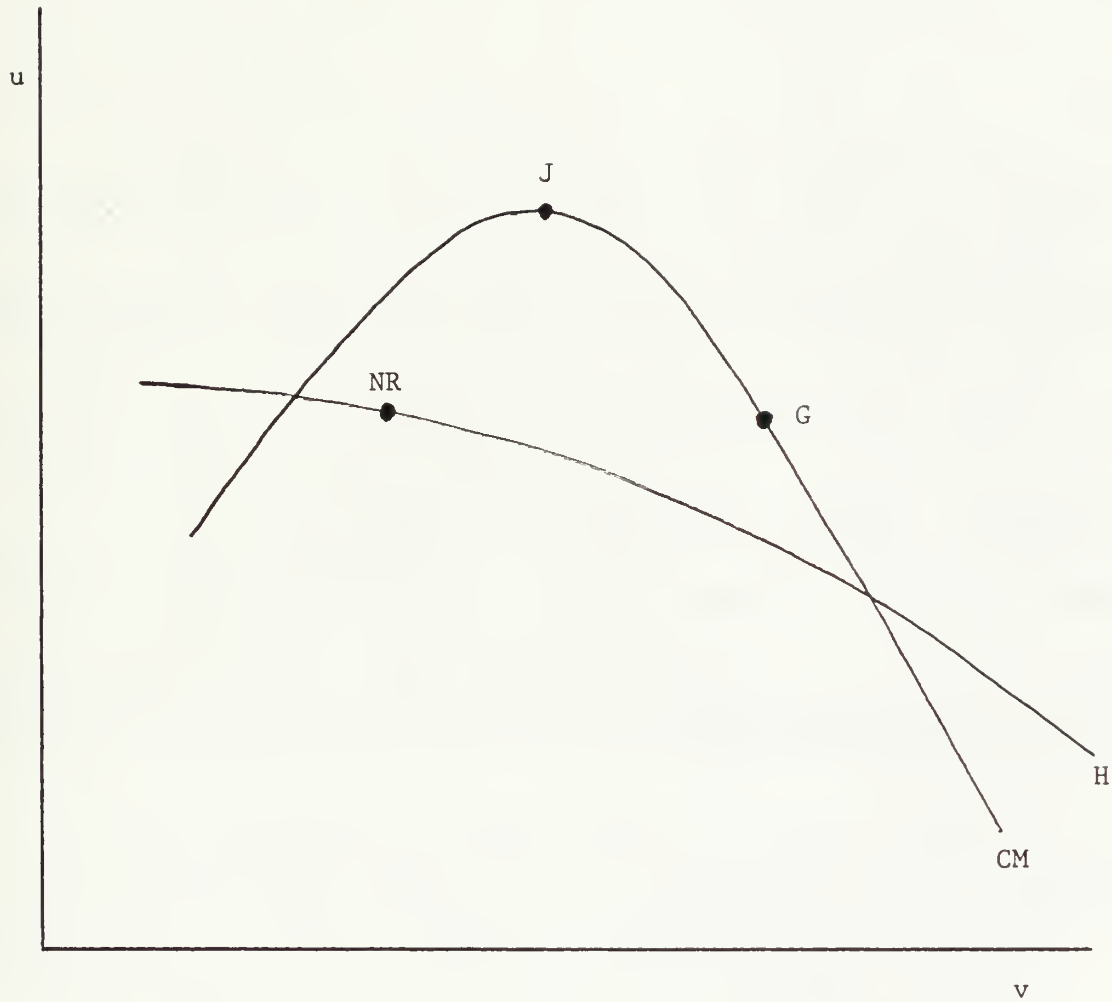

FIGURE 1.

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Footnotes

*Professor of Economics and Ph.D. candidate in Economics respectively. We are indebted to Lanny Arvan for extremely helpful comments on an earlier version of this paper. David Pines also provided some useful criticism. Any errors or shortcomings are, of course, our responsibility.

[1]Pauly's conclusion is in fact too strong in that national redistribution is possibly though not necessarily inefficient. While it is true that a uniform standard makes a national policy more restrictive than local policies, transfers that are feasible under a national program are not always feasible under local redistribution. National redistribution in Pauly's model is therefore inferior to local redistribution on one count but superior on another, making the preferred policy indeterminate.

[2]See Oates (1972) for a statement of the standard position. More recently, Oates analysed equilibrium outcomes in a model similar to Pauly's (see Brown and Oates (1987)).

[3]Brueckner (1988) used a similar approach to analyse local redistribution in the absence of altruism.

[4]See Berglas and Pines (1981) and Berglas (1984).

[5]Recall that a previous assumption on C guarantees an interior solution to (5).

[6]In a mixed-club problem without altruism, Brueckner (1988) shows that $x^a = 0$ must hold in the upper range of possible v values, with $x^b = 0$ holding in the lower range of v values. The present problem exhibits a similar outcome, with nonnegativity constraints on x^a binding for large values of v. It is also possible that x^b may be zero at the CM solution, although the presence of altruism precludes any definite statement. While the Samuelson condition (8) is altered when nonnegativity constraints are binding, this has no effect on any of the results derived below.

[7]It should be noted that in contrast to Pauly's characterization of Pareto-efficiency, the welfare of both the rich and the poor is taken into account in the above discussion (Pauly's Pareto-optimum was defined relative to the nonpoor members of the club).

[8]Since it can be shown that the feasible v's under CM are a subset of the feasible v's under H, the endpoints of the CM frontier lie inside those of the H frontier.

[9]A single member of the family of H configurations (that based on the original distribution of income) is in fact attainable.

[10]The integer problem is again ignored.

[11]It should be noted that the optimization problem may look ill-defined when Q = 0 since the objective function applies to a mixed club. However, the multiplier for constraint (11) assumes the value -1 when Q = 0, so that the U function for mixed clubs drops out of the problem and the first-order conditions reduce to those for the H problem considered earlier.

[12]It should be noted that when Q = 0, the x^1, z, and n variables in the mixed club are in fact undetermined. This problem is handled by setting these variables at Q = 0 equal to their limits as Q approaches zero. Note that the disappearance of mixed clubs means that the multipliers associated with constraints (11) and (12) equal zero when Q = 0.

[13]Note that when (14) is nonnegative for all σ and zero for some σ, then mixed clubs with the given σ can be formed with no gain or loss of resources. In this case, the optimal club configuration is indeterminate, with any combination of mixed and homogeneous clubs being optimal.

[14]To see this, consider the case where $\sigma \geq \theta$. Letting D denote the cost of displacing one a-type, total displacement cost equals D times the number of a's outside mixed clubs, which is $(1-\theta)N - \theta N(1-\sigma)/\sigma$. To find the displacement cost per a-type remaining in mixed clubs, this must be divided by $\theta N(1-\sigma)/\sigma$, which yields $D(\sigma-\theta)/\theta(1-\sigma)$. The derivative of this expression with respect to σ (with D held fixed) evaluated at $\sigma = \theta$ is the second term in (15). A similar argument applies to (16).

[15]This fact about (14), as well those derived below, does not depend on the Q value at which the expression is evaluated.

[16]Note that T is in fact unrestricted in sign. Also, note that the developer does not retain any of the taxes he collects from the a's for himself. The developer could in fact reduce the transfer paid to the b's to eT, with $e \leq 1$, earning an additional $\theta n(1-e)T$ in profit. However, it can be shown that the profit maximizing value of e is unity.

[17]T_z is proportional to $U_z/U_x - V_z/V_x$.

[18]It should be noted that this definition corresponds to the Nash equilibirum analysed by Berglas (1976b) rather than to the strictly competitive equilibrium discussed by Berglas and Pines (1981). The reason is that developers explicitly take into account the utilities offered by other developers rather than viewing the club membership fee as a parametric price schedule.

[19]It should be noted that this discussion (as well as that below) relies on the absence of generalized altruism in that the type-a utility level in an alternative club does not depend on the prevailing type-b utility in the original club configuration. With generalized altruism, by contrast, the k value in an alternative mixed club would depend on the club's own v value as well as the v level in the original configuration. With type-b utilities nonuniform, the profitability of such a club could not be evaluated by referring to the CM frontier, which presumes uniformity. It is easy to see, however, that since the terms in the k

argument that involve β disappear, the CM and H frontiers are appropriate for evaluating alternative clubs in the absence of generalized altruism.

[20]T is positive because the b's reach a higher utility than in a homogenous club in spite of the efficiency loss of joint public consumption. As a result, $x^b + C/n$ must exceed consumption in a homogeneous club, which equals I^b. But since $x^b = I^b - P + T$, it follows that $T > 0$.

[21]To analyse equilibrium, consider the mixed club problem based on (6) and (7) without the constraint $\sigma = \theta$. This problem is different from the PM problem in that the utility level of individuals not accommodated in mixed clubs is not considered. It is easy to show that the zero-profit solution to the mixed-club developer's problem with σ free (under which (21) equals zero) is the same as the solution to this modified planning problem (for given v). Let a new utility frontier (denoted MM) show the u value for this problem at each v. Suppose for the moment that the MM frontier lies everywhere above the CM frontier (indicating that the optimal $\sigma \neq \theta$) and that the CM frontier passes to the northeast of NR. It is clear that for mixed clubs to exist in a free-σ equilibrium, the utility pair they offer must lie on the MM frontier. Moreover, the individuals excluded from mixed clubs must reach the same utility as mixed-club residents in equilibrium. However, since homogenenous-club utilities must equal the NR values, only two MM points are candidates for equilibrium. The MM point with $v = v_{NR}$ is an equilibrium candidate when mixed clubs have σ less than θ, and the MM point with $u = u_{NR}$ is an equilibrium candidate when mixed clubs have σ greater than θ (it can be shown that these points correspond to PM solutions from the general planning problem). Suppose than $\sigma < \theta$ holds along the MM frontier to the northeast of NR, so that the $v = v_{NR}$ point is the only equilibrium candidate. If this point is not Pareto-dominated by any other MM point to the northeast of NR, then it is the equilibrium. However, if Pareto-superior MM points exist (a possibility given that the MM frontier may have upward-sloping segments), then developers can form viable and profitable alternative clubs with $v > v_{NR}$. Since homogeneous clubs cannot offer this high a utility, the equal-utility requirement of equilibrium is violated and equilibrium does not exist. Note that existence would be guaranteed if the MM frontier were to touch the CM frontier to the northeast of NR. In this case (where the optimal σ equals θ), the associated CM configuration is the equilibrium. This discussion can be extended to handle the cases where the optimal σ exceeds θ or where the CM frontier passes below NR.

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